Middle Exam (Discrete Mathematics) (共四頁) 97.11.18

1. (2%) There are ________ Boolean functions with 3 variables.

2. (3%) When written as a sum of minterms (in the variables x and y), $x + \bar{x}y =$

3. (3%) Write $(x + y)(\bar{x} + \bar{y})$ as a sum-of-products in the variables x and y.

4. (4%) Draw a logic gate diagram for the Boolean function $F(x, y, z) = \overline{(x\bar{y})} + x\bar{z}$.

5. (5%) Use a Karnaugh map to minimize the sum-of-products expression

   $xyz + x\bar{y}z + \bar{x}\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z}$.

6. (5%) In the questions below determine whether the proposition is TRUE or FALSE

   1. 1 + 1 = 3 if and only if 2 + 2 = 3.
   2. If it is raining, then it is raining.
   3. If 1 < 0, then 3 = 4.
   4. If 2 + 1 = 3, then 2 = 3 − 1.
   5. If 1 + 1 = 2 or 1 + 1 = 3, then 2 + 2 = 3 and 2 + 2 = 4.

7. (4%) 小明跟爸爸說"如果好好用功，則給我糖吃"；
   爸爸解讀小明意思爲：「如果給你糖吃，則你會好好用功」；
   是否正確？請說明之。

8. (3%) Prove that $(q \wedge (p \rightarrow \neg q)) \rightarrow \neg p$ is a tautology using propositional equivalence and the laws of logic.

9. (5%) Determine whether the following argument is valid. (必須寫出推導過程)
   If you are in the tennis tournament, you will meet Ed.

If you aren't in the tennis tournament or if you aren't in the play, you won't meet Kelly.
You meet Kelly or you don't meet Ed.
It is false that you are in the tennis tournament and in the play.
Therefore, you are in the tennis tournament.

10. (5%) In the questions below mark each statement TRUE or FALSE. Assume that the statement applies to all sets.

1. $A - (B - C) = (A - B) - C$.
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
3. $\overline{A \cup \overline{B}} \cup \overline{A} = \overline{A}$
4. If $A \cup C = B \cup C$, then $A = B$.
5. If $A \cap C = B \cap C$, then $A = B$.

11. (2%) Give an example of a function f: $\mathbf{Z} \to \mathbf{Z}$ that is onto $\mathbf{Z}$ but not 1-1.

12. (1%). Suppose f: $\mathbf{N} \to \mathbf{N}$ has the rule $f(n) = 4n + 1$. Determine whether f is 1-1.

13. (1%).Suppose f: $\mathbf{N} \to \mathbf{N}$ has the rule $f(n) = 4n + 1$. Determine whether f is onto $\mathbf{N}$.

14 (1%). Suppose f: $\mathbf{Z} \to \mathbf{Z}$ has the rule $f(n) = 3n^2 - 1$. Determine whether f is 1-1.

15. (1%). Suppose f: $\mathbf{Z} \to \mathbf{Z}$ has the rule $f(n) = 3n - 1$. Determine whether f is onto $\mathbf{Z}$.

16. (1%). Suppose f: $\mathbf{N} \to \mathbf{N}$ has the rule $f(n) = 3n^2 - 1$. Determine whether f is 1-1.

17. (4%) In the questions below suppose $g : A \to B$ and f: $B \to C$ where $A = \{1,2,3,4\}$, $B = \{a,b,c\}$, $C = \{2,8,10\}$, and g and f are defined by $g = \{(1,b),(2,a),(3,b),(4,a)\}$ and $f = \{(a,8),(b,10),(c,2)\}$.

1. Find $f \circ g$.
2. Find f^{-1}.
3. Find $f \circ f^{-1}$.

4. Explain why g^{-1} is not a function.

18. (3%). Describe an algorithm that takes a list of n integers ($n \geq 1$) and finds the average of the largest and smallest integers in the list.

19. (2%) Use the definition of big-oh to prove that $\dfrac{6n+4n^5-4}{7n^2-3}$ is $O(n^3)$.

20. (2%). Find the best big-O function for $\dfrac{x^3+7x}{3x+1}$.

21. (2%) Find –88 **mod 13.**

22. (2%) Convert $(11101)_2$ to base 10.

23. (3%) Prove: if n is an integer that is not a multiple of 3, then $n^2 \equiv 1 \bmod 3$.

24. (3%). Solve the linear congruence $2x \equiv 5 \pmod 9$.

25. (3%). Use the Principle of Mathematical Induction to prove that $n^3 > n^2 + 3$ for all $n \geq 2$.

26. (3%) Use the Principle of Mathematical Induction to prove that $4 \mid (9^n - 5^n)$ for all $n \geq 0$.

27. (3%) Find $f(2)$ and $f(3)$ if $f(n) = f(n-1)/f(n-2)$, $f(0) = 2$, $f(1) = 5$.

28. (4%) In the questions below give a recursive definition (with initial condition(s)) of $\{a_n\}$ ($n = 1,2,3,\ldots$).

1. $a_n = 2^n$.
2. $a_n = 3n - 5$.
3. $a_n = (n + 1)/3$.
4. $a_n = \sqrt{2}$.

29. (3%) Verify that the following program segment:

if $x \leq y$ **then**

$max := y$

else

$max := x.$

is correct with respect to the initial assertion **T** and the final assertion $(x \leq y \wedge max = y) \vee (x > y \wedge max = x)$.

30. (3%). In the questions below consider all bit strings of length 12.

1. How many begin with 110?
2. How many begin with 11 or end with 10?
3. How many have exactly four 1s and none of these 1s are adjacent to each other?

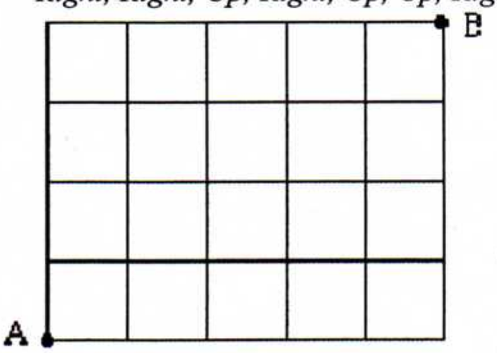
31. (4%). There are 15 workstations and 10 servers. A cable can be used to connect a workstation to a server. Two goals: (1) We want to guarantee that any time any set of 10 or fewer workstations can simultaneously access different servers. (2) We want to guarantee that any time any set of 5 or fewer workstations can simultaneously access different servers. What is the minimum number of direct connections needed to achieve these goals?

32. (2%). Suppose $|A| = 10$ and $|B| = 4$. Find the number of 1-1 functions $f: A \rightarrow B$.

33. (4%). The figure shows a 4-block by 5-block grid of streets.

Find the number of ways in which you can go from point A to point B, where at each stage you can only go right or up. (You are not allowed to go left or down.) For example, one allowable route from A to B is:

Right, Right, Up, Right, Up, Up, Right, Right, Up.



34.(2%) Find the next two largest permutation in lexicographic order after 52143.

35. (2%) Find the next four largest 4-combinations of the set $\{1,2,3,4,5,6,7,8\}$ in lexicographic order after $\{1,2,3,5\}$.