

(滿分 100)

// 所有問題不能只寫答案(例如: Yes/ No), 必須加予說明, 否則不計分

// 答案卷 請寫上姓名、學號

1. (4%) 為何本課程內容稱為"離散數學" (discrete mathematics) ?
2. (4%) 用真值表表示 $p \rightarrow q$ 之所有結果。
3. (4%) Determine whether each of these conditional statements is true or false. (說明原因)
 - a) If $2 + 2 = 2$, then $2 + 3 = 6$.
 - b) If $2 + 1 = 3$, then $2 + 2 = 5$.
 - c) If $1 + 1 = 3$, then $2 + 2 = 5$.
 - d) If birds can fly, then $1 + 1 = 3$.
4. (4%) Determine the result of $\neg (p \rightarrow q)$.
5. (4%)
 - 若小明吃冰淇淋, 就會吃熱狗。
 - 若天氣熱, 小明就會吃冰淇淋。
 - 若看電影, 小明就會吃爆米花。
 - 小明若沒吃冰, 就會吃熱狗。
 - 若小明跟小花在一起, 就會吃一樣的東西。
 - 若小花吃冰, 則不會吃冰淇淋。
 - 若小花吃爆米花, 就不吃熱狗。

⇒ 今天小明與小花一起去看電影, 請問他們會吃哪些東西?
6. (4%) 小明對小花說: 若我買鑽戒給你, 就表示喜歡你。
小花說: 那你沒買鑽戒給我, 就表示不喜歡我。
請問: 小花說法邏輯上是否成立? (原因為何?)
7. (4%) Prove $\sqrt{2}$ is irrational (無理數) by giving a proof by contradiction.
8. (4%) Two players, taking turns removing one, two, or three stones at a time from a pile (一堆) with 15 stones. The person who removes the last stone wins the game. Show that the first player can win the game no matter what the second player does.
9. (4%) Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be such that $f(x) = x+1$. Is f invertible, and if it is, what is its inverse?
10. (4%) How many functions are there for a Boolean function with degree two?

(4%) Find the sum-of-products: $F(x, y, z) = (x + y)\bar{z}$

(4%) Design a voting circuit for majority voting with three inputs.

(4%) If f and $f \circ g$ are onto, does it follow that g is onto? (舉例說明)

(4%) Using K-maps to minimize the boolean function.

a) $x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z$

b) $xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z$

(4%) How many different Boolean functions $F(x, y, z)$ are there such that $\bar{y}, \bar{z} = F(x, y, z)$ for all values of the Boolean variables $x, y,$ and z . (需說明原因)

(3%) Determine whether each of these statements is true or false.

a) \emptyset , b) $\emptyset \in \{0\}$, c) $\emptyset \subset \{0\}$, d) $\{0\} \subset \{0\}$,

(4%) Determine whether the function $f: Z \times Z \rightarrow Z$ is onto if

a) $f(m, n) = m + n$ b) $f(m, n) = m^2 + n^2$ c) $f(m, n) = m$ d) $f(m, n) = |n|$ (必須說)

(4%) Design a half adder (沒前一級的進位) circuit for two input x, y .

(4%) 使用下列兩公式計算各題總合 (需使用下列公式且列式才給分)

$$\sum_{j=1}^n (a_j - a_{j-1}) = a_n - a_0 ; \quad k^2 - (k-1)^2 = 2k - 1$$

(2k-1) b) $\sum_{k=1}^n k$

(4%) 以二進位設計一個四捨五入最簡電路(四個 bits, 如下表)

TABLE 1

<i>Digit</i>	<i>w</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>F</i>
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1