

國立台南大學 九十七學年度第一學期 期中考試 線性代數 試卷 (數位二)

_____年_____班 學號:_____姓名:_____ 共 1 張 2 面

1. (6%) Reduced Echelon Form, Determine whether the following matrices are in reduced echelon form. If a matrix is not in reduced echelon form, give a reason.

(a)
$$\begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 1 & 2 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

2. (10%) Use the method of Gauss-Jordan elimination to solve (if possible) the systems of equations.

$$x_1 + 4x_2 + x_3 = 2$$

$$x_1 + 2x_2 - x_3 = 0$$

$$2x_1 + 6x_2 = 3$$

3. (15%)
$$A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & -3 & -5 \\ 0 & -2 & -6 \end{bmatrix}$$

- (a) Describe the solution set of $Ax = 0$ in parametric vector form(參數)

(b) Give a geometric description of the solution set you find in (a),
(Hint: give a graph in the vector space)

(c) Describe the solution set of $Ax = \begin{bmatrix} 1 \\ 3 \\ -10 \end{bmatrix}$ in parametric vector form (參數)

4. (5%) Solve the following equation for x.

$$\begin{vmatrix} 2x & -3 \\ x-1 & x+2 \end{vmatrix} = 1$$

5. (10%) Prove that $|A^{-1}A'A| = |A|$,

6. (12%) If A, B are a 3x3 matrices with $|A| = 4$, $|B| = -3$, assume A^{-1} exists, compute the following determinants.

(a) $|AA'|$,

(b) $|2B|$,

(c) $|A^{-1}A'A|$

(d) $|5A^{-1}B'|$,

7. (10%) Show that if $A' = A^{-1}$, then $|A| = \pm 1$.

8. (12%) Determine the values of λ for which the following system of equations has nontrivial solutions.

Find the solutions for each value of λ

$$(1 - \lambda)x_1 + 6x_2 = 0$$

$$5x_1 + (2 - \lambda)x_2 = 0$$

9. (10%)
$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 2x_1 - x_2 + 3x_3 &= 5 \\ 4x_1 + 5x_2 + x_3 &= 3 \end{aligned}$$
 Solve the system of equations using Cramer's rule.

10. (15%)
$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 1 \\ 5 & 2 & 0 \end{bmatrix}$$

- (a) Describe the elementary row operations which transform A into I
- (b) Find A^{-1}
- (c) Suppose $BC = A$, Find C^{-1}

